

Fibonacci sequence

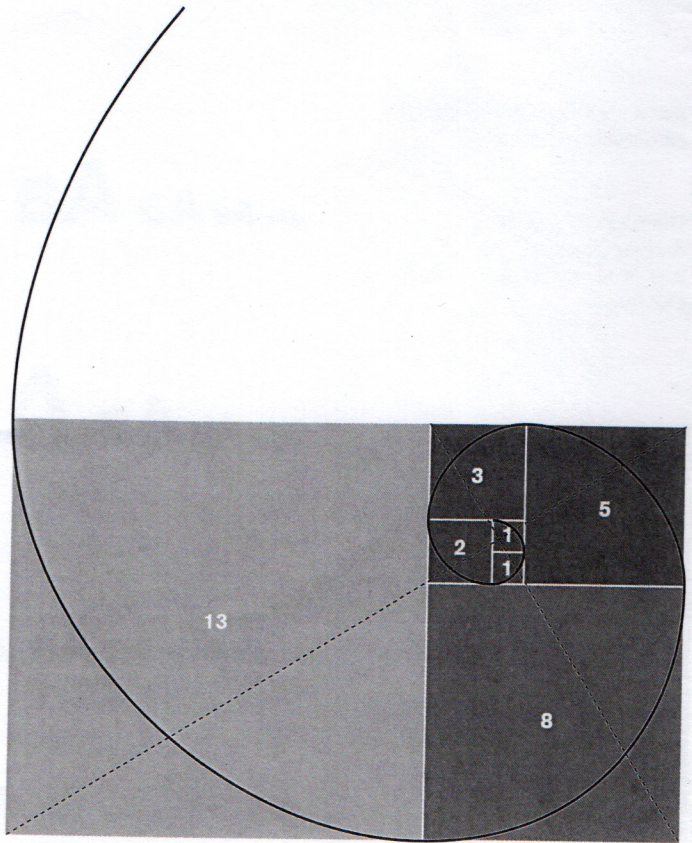
Another useful model when considering proportions is the Fibonacci sequence. Named for Italian mathematician Leonardo Fibonacci (c.1170–1240), a Fibonacci sequence describes a sequence in which each number is the sum of the two preceding numbers:

- 0
- 1
- 1 [1+0]
- 2 [1+1]
- 3 [1+2]
- 5 [2+3]
- 8 [3+5]
- 13 [5+8]
- 21 [8+13]
- 34 [13+21]
- ...

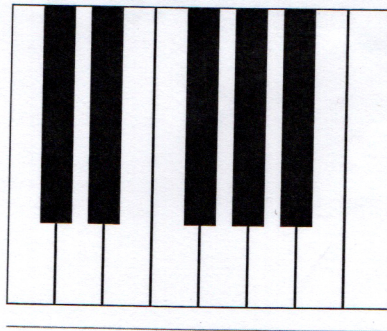
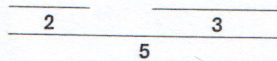
As the numbers in a Fibonacci sequence increase, the proportion between any two numbers very closely approximates the proportion in a golden section (1:1.618). For example, 21:34 approximately equals 1:1.618. Nature is full of examples of the Fibonacci sequence and the golden section, from the intervals of branches on a tree to the shell of a chambered nautilus.

Fibonacci's sequence always began with 1 but the proportion between any two numbers remains constant when the sequence is multiplied:

| | | |
|-----|-----|-----|
| 0 | 0 | 0 |
| 2 | 3 | 4 |
| 2 | 3 | 4 |
| 4 | 6 | 8 |
| 6 | 9 | 12 |
| 10 | 15 | 20 |
| 16 | 24 | 32 |
| 26 | 39 | 52 |
| 42 | 63 | 84 |
| 68 | 102 | 136 |
| ... | ... | ... |



Above, a spiral describing a Fibonacci series (and the growth of a chambered nautilus). The red rectangle on the upper right approximates a golden section. As each square in the sequence is added, the orientation of the golden section changes from vertical to horizontal.



Left, one of the many examples of a Fibonacci sequence is the musical octave as seen on a piano—eight white keys and five black keys (separated into a group of two and a group of three).